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Time-optimal operation of the hybrid Formula 1 power unit

Dr. Pol Duhr, Marc Neumann, Giona Fieni 27th January 2023

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People



Dr. Pol Duhr PhD from 2018 to 2022

First part: Research of the past 4 years



Marc Neumann PhD student



Giona Fieni PhD student

Second part: Outlook on future research

Project overview





Since 2012: Collaboration with the department "PU Performance and Control Strategies"

3 PhDs completed2 PhDs ongoing

More than 20 bachelor, semester and master theses

Publications 9 journal papers 5 conference papers

10 optimization tools delivered

Weekly videocalls for close support of the industrial partner, plus trips to Maranello

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Formula 1 racing

Bahrain Circuit $N_{\text{laps}} = 57 \implies 308 \text{ km}$



Race time \approx 95 min **Lap time** \approx 95 s





Examples from 2022:

POS	DRIVER	C	AR	Q3		
1	Carlos Sainz	F	ERRARI	1:40.983	072 a	
2	Max Verstappen		ED BULL RACING RBI	PT 1:41.055	. 0 / 2 S	
		POS	DRIVER	CAR	TIME/RETIRED	
		1	Max Verstappen	RED BULL RACING RBPT	1:24:19.293	
		2	Charles Leclerc	FERRARI	+0.549s	
					4	

Formula 1 racing

Bahrain Circuit $N_{\text{laps}} = 57 \implies 308 \text{ km}$



Race time \approx 95 min **Lap time** \approx 95 s





Development progress: Measured in **milliseconds**!

Maximize powertrain potential



Hybrid Formula 1 power unit



Minimize lap time



Hybrid Formula 1 power unit



Minimize lap time



Energy is limited → operate PU efficiently!

Regulations allow automatic energy management

Hybrid Formula 1 power unit



Minimize lap time



Energy is limited
→ operate PU efficiently!

Regulations allow automatic energy management

Optimal energy management





Offline optimization Optimal energy management



Optimal P_{e} , P_{k} , P_{h} , ... **at each point** on track?

Not trivial!

Minimum-lap-time optimization



Grip limit model



Velocity v(s) not fixed, optimization variable!

Constraint on velocity: $v(s) \le v_{\max}(s) \quad \forall s$

Second-order cone program

S. Ebbesen, M. Salazar, P. Elbert, C. Bussi, and C. H. Onder. Time-optimal control strategies for a hybrid electric race car. IEEE Transactions on Control Systems Technology, 26(1):233–247, 2018.

 $f^T \cdot x$ $\min_{x \in \mathbb{R}^n}$ subject to $F \cdot x = g$ $H \cdot x \leq k$ $||A_i \cdot x + b_i||_2 \le c_i^T \cdot x + d, i = 1, ..., m$ second-order cone constraints

linear objective linear equality constraints linear inequality constraints





Convex optimization problem: \rightarrow Very efficient numerical algorithms \rightarrow Optimality guarantees





Gearshift optimization









Minimum-race-time energy allocation



P. Duhr, G. Christodoulou, C. Balerna, M. Salazar, A. Cerofolini, and C. H. Onder. **Time-optimal gearshift and energy management strategies for a hybrid electric race car.** Applied Energy, 282:115980, 2020.

Drivetrain model



Overall transmission ratio: $\omega_e = \Gamma_g \cdot v$ with $g \in \{1, ..., 8\}$

Research questions

Selected gear defines engine speed.

Given that the engine speed has an impact on

- the achievable propulsive power
- the electric recuperation

→ Interactions between **optimal energy management** and **optimal gear selection**?

→ **Sub-optimality** of a simple heuristic gearshift rule similar to the driver's behaviour?

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Engine model



Validation plots:





Engine model

Engine power:

 $P_{\rm e} = e(\omega_{\rm e}) \cdot P_{\rm f} + P_{\rm fric}(\omega_{\rm e}) + P_{\rm pump}(\omega_{\rm e}, P_{\rm f}, r_{\rm wg})$



Pumping power:

- Engine can be seen as a volumetric pump
- Opening the waste-gate reduces exhaust manifold pressure
 → larger pressure difference → pumping power increases

$$P_{\text{pump}}(\omega_{\text{e}}, P_{\text{f}}, r_{\text{wg}}) = \left(\frac{C_{1}}{\eta_{\text{vol}}(\omega_{\text{e}})} - C_{2} \cdot \omega_{\text{e}} \cdot (\alpha_{\text{wg,1}} \cdot r_{\text{wg}} + \alpha_{\text{wg,0}})\right) \cdot P_{\text{f}}$$





 $r_{wg} \in [-1, 0]$

MGU-H model

Assumption: MGU-H is only operated in generator mode.

$$P_{\rm h} = \eta_{\rm h}(\omega_{\rm e}) \cdot P_{\rm f} \cdot r_{\rm wg} \le \mathbf{0}$$



0

 \rightarrow open



Observations – Trade-offs!

Waste-gate open

 \rightarrow small increase in engine power

but

 \rightarrow MGU-H recuperation becomes zero

- High engine speed
 → engine efficiency decreases
 but
 - \rightarrow more MGU-H recuperation

Optimization problem

Lap time minimization min

 $\min \int_0^T \mathrm{d}t$

Ideally:

Optimize everything at once

$$\omega_e = \Gamma_g \cdot v$$
Terms $e(\omega_e) \cdot P_f$ etc.Not convexInteger variable

Problem:

Would lead to a mixed-integer non-linear program (MINLP) \rightarrow computationally very expensive, convergence and optimality issues

Solution:

- Iterative scheme
- Separate optimization of integer and continuous variables

Iterative optimization scheme



 \bar{v} $\bar{P}_{f}, \bar{P}_{k}, \bar{r}_{wg}, \bar{P}_{p}$ Costates λ (damped)

Input:	Sequential gearshift command $u_g(k) \in \{-1, 0, 1\}$
State:	Selected gear $\in \{1,, 8\}$ $g(k+1) = g(k) + u_g(k)$
Stage cost:	Hamiltonian $\widetilde{H}(\boldsymbol{g}(\boldsymbol{k}), \bar{v}, \bar{P}_{f}, \bar{P}_{k}, \bar{r}_{wg}, \bar{P}_{p}, \lambda)$ \rightarrow captures optimal trade-offs \rightarrow propulsive power vs. fuel vs. electric consumption

$$P_{\rm e} = \frac{e(\omega_{\rm e})}{P_{\rm f}} \cdot P_{\rm f} + \frac{P_{\rm fric}(\omega_{\rm e})}{P_{\rm fric}} + P_{\rm pump}(\omega_{\rm e}, P_{\rm f}, r_{\rm wg})$$
$$P_{\rm h} = \frac{\eta_{\rm h}(\omega_{\rm e})}{\eta_{\rm h}(\omega_{\rm e})} \cdot P_{\rm f} \cdot r_{\rm wg}$$

Second-order cone program



Iterative optimization scheme



Iterative optimization scheme





Results – Bahrain Circuit

- Compare optimal gearshift strategy for: $\Delta E_{\rm b} = 0 \text{ MJ}$ (charge-sustained) $\Delta E_{\rm b} = -2 \text{ MJ}$ (strong discharge)
- Sub-optimality of a heuristic gearshift rule



Computational time: 10 iterations, 90 seconds

Optimal trajectories



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Optimal trajectories



Optimal trajectories



Heuristic gearshift strategy



- "Real world": **LEDs** on the steering wheel tell the driver when to upshift
- → Threshold-based upshift rule $\omega_e \ge \omega_{e,upshift} \rightarrow u_g = +1$



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Sweep in upshift engine speed





Gearshift optimization









Minimum-race-time energy allocation



P. Duhr, A. Sandeep, A. Cerofolini, and C. H. Onder. **Convex performance envelope for minimum lap time energy management of race cars.** IEEE Transactions on Vehicular Technology, 71(8):8280–8295, 2022.

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g-g diagram





Boundary of feasible combinations {acceleration, velocity}

Performance envelope

Research question

Given that in minimum-lap-time optimization frameworks

- a model for the car's grip limit is required
- the simple maximum velocity profile approach has some drawbacks
- convex optimization is computationally very efficient

→ Convex model for performance envelope, using second-order cone constraints?

Vehicle dynamics model



Quasi steady-state approximation

 $v(s) = \frac{\mathrm{d}s}{\mathrm{d}t}(s)$

$$a_{\text{long}}(s) = \frac{dv}{dt}(s)$$

$$a_{\text{lat}}(s) = \frac{v^2}{R(s)} = v^2 \cdot \kappa(s) \quad \propto \text{ total lateral tire force}$$

$$a_p(s) = \frac{1}{m} \cdot F_p \quad \propto \text{ total longitudinal tire force}$$

Optimization variables

Second-order cone program:

Linear (in-)equality constraints Second-order cone constraints

Optimization variable Normalized kinetic energy

$$\tilde{E}_{\rm kin}(s) = \frac{1}{2} \cdot v(s)^2$$

Longitudinal dynamics (Newton's law)

$$\frac{\mathrm{d}}{\mathrm{d}s}\tilde{E}_{\mathrm{kin}}(s) = \frac{1}{m} \cdot \left(F_{\mathrm{p}}(s) - F_{\mathrm{d}}(s)\right)$$



Link between a_{p} , a_{lat} and longitudinal dynamics:

$$a_{p}(s) = \frac{1}{m} \cdot F_{p}(s) \qquad \longrightarrow \qquad a_{p}(s) = \frac{1}{m} \cdot F_{p}(s)$$

$$a_{lat}(s) = v^{2} \cdot \kappa(s) \qquad \longrightarrow \qquad a_{lat}(s) = 2 \cdot \tilde{E}_{kin}(s) \cdot \kappa(s)$$
Linear equality constraints

Performance envelope model



Ellipse equation:

$$\frac{a_{\rm p}^2(s)}{a_{\rm p,max}^2} + \frac{a_{\rm lat}^2(s)}{a_{\rm lat,max}^2} \le 1 \qquad |\cdot a_{\rm p,max}^2|$$

$$\Leftrightarrow \quad a_{\rm p}^2(s) + \left(\frac{a_{\rm p,max}}{a_{\rm lat,max}} \cdot a_{\rm lat}(s)\right)^2 \le a_{\rm p,max}^2$$

$$\Leftrightarrow \quad \left\| \frac{a_p(s)}{a_{p,\max}} \cdot a_{lat}(s) \right\|_2 \le a_{p,\max}$$

Second-order cone constraint



Performance envelope model



Vehicles behave differently under acceleration vs. deceleration

→ Two half ellipses → One for $a_p \ge 0$, one for $a_p \le 0$

$$a_{p}(s) = a_{p}^{+}(s) + a_{p}^{-}(s)$$
$$a_{p}^{+}(s) \ge 0$$
$$a_{p}^{-}(s) \le 0$$



Performance envelope model



Velocity dependency:For constant $a_{lat,max}(s) = c_1 \cdot v^2(s) + c_2 \cdot v(s) + c_3$ $a_{p,max}^+(s) = a_{p,max}(s)$ $\Rightarrow a_{lat,max}(s) = 2c_1 \cdot \tilde{E}_{kin}(s) + c_2 \cdot v(s) + c_3$ $a_{p,max}^-(s) = a_{p,max}(s)$

For convexity: $a_{p,\max}^+(s) = r^+ \cdot a_{lat,\max}(s)$ $a_{p,\max}^-(s) = r^- \cdot a_{lat,\max}(s)$

Linear constraints $a_{p}(s) = a_{p}^{+}(s) + a_{p}^{-}(s)$ $a_{p}^{+}(s) \ge 0$ $a_{p}^{-}(s) \le 0$

Secondorder cone constraints

$$\begin{vmatrix} a_{\mathrm{p}}^{+}(s) \\ r^{+} \cdot a_{\mathrm{lat}}(s) \end{vmatrix} _{2} \leq r^{+} \cdot (2 c_{1} \cdot \widetilde{E}_{\mathrm{kin}}(s) + c_{2} \cdot v(s) + c_{3})$$
$$\begin{vmatrix} a_{\mathrm{p}}^{-}(s) \\ r^{-} \cdot a_{\mathrm{lat}}(s) \end{vmatrix} _{2} \leq r^{-} \cdot (2 c_{1} \cdot \widetilde{E}_{\mathrm{kin}}(s) + c_{2} \cdot v(s) + c_{3}) \end{vmatrix}$$

Performance envelope fit



Optimization results



Second-order cone program

Silverstone Circuit



Computational time: 1 second

Optimization result vs. measurement



Performance envelope 3D





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Scaling the performance envelope







Gearshift optimization









Minimum-race-time energy allocation



P. Duhr, D. Buccheri, C. Balerna, A. Cerofolini, and C. H. Onder. **Minimum-race-time energy allocation strategies for the hybrid-electric Formula 1 power unit.** IEEE Transactions on Vehicular Technology, 2023.

Race energy allocation

Team's objective Finish race before competitors



Engineering objective Complete **race** in **minimum time**



Before the race



How much fuel to put in the tank?

Trade-off! More fuel → more energy for PU but → heavier car

Research questions

Assuming that

- the objective is to minimize race time
- within each lap, the allocated energy is used optimally

Optimal fuel and battery energy allocation for each lap?
Optimal fuel load at the start of the race?

Race optimization framework



Lap time map



- Fuel consumed during lap $\Delta E_{\rm f}$
- Battery charge/discharge during lap ΔE_{b}
- Mass of the car

- → modeled by scaling the **performance envelope**
- Battery energy at the start of the lap $E_{b,init} \rightarrow$ only influences lap time when close to bounds

m



Lap time map

 $T_{\text{lap}} \approx \mathcal{M}(\Delta E_{\text{f}}, \Delta E_{\text{b}}, E_{\text{b,init}}, m) = T_{\text{lap,nom}}(\Delta E_{\text{b}}, \Delta E_{\text{f}}, m) + \Delta T_{\text{lap,b}}(E_{\text{b,init}}, \Delta E_{\text{b}})$



Optimization problem

Objective	$\min\sum_{i=1}^{N_{\text{laps}}} T_{\text{lap}}[i]$	Car mass	$m[i] = m_{\text{car}} + m_{\text{f}} - \frac{E_{\text{f}}[i]}{H_{\text{lhv}}}$
Inputs	$\begin{aligned} \Delta E_{\rm f}[i], \Delta E_{\rm b}[i], T_{\rm lap}[i] \\ \Delta E_{\rm f}[i] &\geq 0 \\ T_{\rm lap}[i] &\geq \mathcal{M}(\Delta E_{\rm f}[i], \Delta E_{\rm b}[i], m[i], E_{\rm b}[i]) \end{aligned}$	Battery constraints	$\begin{split} E_{\rm b}[1] &= 4 \text{ MJ} \\ E_{\rm b}[N_{\rm laps} + 1] \geq 0 \text{ MJ} \\ E_{\rm b}[i] \geq 0 \text{ MJ}, E_{\rm b}[i] \leq 4 \text{ MJ} \end{split}$
State dynamics	$E_{f}[i+1] = E_{f}[i] + \Delta E_{f}[i]$ $E_{b}[i+1] = E_{b}[i] + \Delta E_{b}[i]$	Fuel constraints	$E_{\rm f}[1] = 0$ $E_{\rm f}[N_{\rm laps} + 1] \le m_{\rm f} \cdot H_{\rm lhv}$

- Parameter: fuel load $m_{\rm f}$
- Lap time: relaxed to inequality
- Linear equations except for lap time maps (neural networks)
- \rightarrow Non-linear program, solved in 1 second



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Optimal solution vs. simple heuristic



Optimal race fuel load



Summary

- 1) Energy management has an impact on optimal gearshift strategy
- 2) Performance envelope is a threedimensional tube and can be represented by convex constraints
- 3) Optimal energy allocation can gain several seconds over a race distance

Outlook







Optimal Energy Management of the 2026 F1 Power Unit

Marc Neumann, Giona Fieni, Dr. Pol Duhr Prof. Dr. Christopher Onder

Formula 1 Power Unit – Models



ICE power P_e



- MGU-K power P_k
- Brake power *P*_{brk}
- Waste-gate position u_{wg}
 - MGU-H power $P_{\rm h}$
 - Engine back-pressure
- Fuel mass flow $\dot{m}_{\rm f}$

Low-Level Model

- MGU-K power P_k
- MGU-H power P_h
- Waste-gate position u_{wg}
- Throttle position $u_{\rm th}$
- Spark advance u_{sa}
- Cylinder deactivation Ψ_e
- Brake power P_{brk}
- Engaged gear Γ

Offline Optimization – Low Level



Offline Optimization – Low Level





C. Balerna, M.-P. Neumann, N. Robuschi, P. Duhr, A. Cerofolini, V. Ravaglioli, and C. Onder, "Time-optimal low-level control and gearshift strategies for the Formula 1 hybrid electric powertrain", *Energies,* vol. 14, p. 171, 2021.

Formula 1 Power Unit – Fuel Regulations



- 5.4.3 Fuel energy flow must not exceed 3000MJ/h.
- 5.4.4 Below 10500rpm the fuel energy flow must not exceed EF(MJ/h)=0.27*N(rpm)+ 165

SUSTAINABILITY STRATEGY

16.3 Fuel properties

The only fuel permitted is petrol having the following characteristics:

Property	Units	Min	Max	Test Method
RON		95.0 ⁽¹⁾	102.0(1)	ISO 5164/ ASTM D2699
Sensitivity (RON-MON)			15.0 ⁽¹⁾	ISO 5164/ ASTM D2699 ISO 5163/ ASTM D2700
LHV	MJ/kg	38.0	41.0	GC
Density (at 15°C)	kg/m³	720.0	785.0	ISO 12185/ ASTM D4052
Methanol ⁽²⁾	% v/v		3.0	EN 1601 or EN 13132 or EN ISO 22854
Oxygen	wt%	6.70	7.10	Elemental Analysis
Nitrogen	mg/kg		500	ASTM D 5762
Benzene	wt%		1	GCMS
DVPE	kPa	45	68	EN130161
Lead	mg/l		5	ASTM D 3237 or ICPOES
Manganese	mg/l		2	ASTM D 3831 or ICPOES
Metals (excluding alkali metals)	mg/l		5	ICPOES



Optimal Energy Management of the 2026 F1 Power Unit

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Formula 1 power unit 2026 - challenges



No MGUH:

- Less energy recuperation is possible
- Turbo lag becomes relevant

Engine and MGUK balance:

 Power split between engine and MGUK is very different

Battery and MGUK:

- Same battery
- Battery discharge is 3x faster

Power unit operation without MGUH



No MGUH:

- Less energy recuperation is possible
- Turbo lag becomes relevant

Power unit operation without MGU-H





- The turbo lag becomes a relevant effect
- The waste-gate will play a different role
- Predictive information can be crucial → MPC?

Engine and MGUK balance



Engine and MGUK balance:

 Power split between engine and MGUK is very different

Engine and MGUK balance



- Recuperation does not take place only during braking phases
- The motor can be operated at 100% power with MGUK in full recuperation to recharge the battery



- Since the battery is depleted quicker than in 2023, the recuperation phase is prolonged
 The coasting phase can be quite long!
- Using additional battery energy there, the pilot can count on 700 kW extra for a few seconds to perform an overtake!

Battery and MGU-K



Battery and MGUK:

- Same battery
- Battery discharge is 3*x* faster

Battery and MGU-K

Straight without overtake



Example: overtake maneuver

Assume that for an overtake on a longer straight we need 3s extra MGU-K boost time. This means that the <u>extra</u> energy depleted from the battery is:

2023:
$$120 \ kW = 0.12 \frac{MJ}{s} \rightarrow E_{\text{overtake}} = 0.12 \frac{MJ}{s} * 3s = 0.48 \ MJ \rightarrow 9\%$$
 of the battery energy

2026:
$$350 \ kW = 0.35 \frac{MJ}{s} \rightarrow E_{\text{overtake}} = 0.35 \frac{MJ}{s} * 3s = 1.05 \ M$$

 \rightarrow 26.5% of the battery energy Only for the overtake!

= 17.5% of the battery energy

= 35% of the battery energy



The battery can be emptied very easily.

Consequences for 2026:

• Sufficient energy must be saved to defend the new position!

